Electromagnetic Wave Diffraction on the Conducting Thin Screen Placed on the Isotropic and Anisotropic Media Interface

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Abstract—The over-determined boundary value problem method in the diffraction electromagnetic waves theory is extended to the case of anisotropic media. The solvability conditions of the over-determined boundary value problems for Maxwell equations set in the anisotropic semi-space are obtained in the case of one-axis anisotropy. The representations of solutions of Maxwell equations set by traces of tangential components of the field on the boundary of domain are constructed. The problem on reflection and refraction of the electromagnetic wave on the isotropic and anisotropic media interface is considered. The integral equation is obtained to determine field perturbation of conducting thin screen placed at the media interface.

1. INTRODUCTION

The diffraction problems for the electromagnetic waves on the conducting thin screens belong to classic problems of electrodynamics [1]. By potentials of special form such problems are reduced to integral equations either with weak singularity or to hyper-singular equations. The over-determined boundary value problem method [2] offers to pass from boundary value problems for Helmholtz equation in two-dimensional case or for the Maxwell equations set in the general case to equivalent to them integral equations. By this fundamental solutions or Green functions for the layered medium are not used. In the works [3, 4] the integral equations of different types were obtained to which the electromagnetic wave diffraction problem on the thin screen in isotropic medium is reduced. In the present work, the case is considered when the conducting thin screen is placed on the isotropic and anisotropic media interface.

2. OVER-DETERMINED BOUNDARY VALUE PROBLEM IN THE CASE OF ANISOTROPIC MEDIA

Consider the Maxwell equations set for complex amplitudes $E$ and $H$ of the electric and magnetic vectors of the electromagnetic field harmonically dependent on time
\[
\text{rot } H = i\omega \varepsilon E, \quad \text{rot } E = -i\omega \mu H
\]
in the upper half-space $z > 0$. Let the traces of tangent components $E$ and $H$ be given on the plane $z = 0$ and
\[
[z_0, E](x, y, 0) = e(x, y), \quad [z_0, H](x, y, 0) = h(x, y).
\]
This problem is the over-determined one because boundary functions in conditions (2) cannot be given arbitrary.

We call the solution $E$, $H$ of Maxwell equations set in the domain $z > 0$ outgoing into a half-space if each of its components is the distribution of slow growth not containing harmonics which transfer the energy on infinity (the radiation condition) and the traces (2) are defined correctly in classic sense or in generalized sense.

Let $k$ be a wave number, $k^2 = \omega^2 \mu \varepsilon$. Denote
\[
\gamma(\xi, \eta) = \left\{\xi^2 + \eta^2 \geq k^2 : i \sqrt{\xi^2 + \eta^2 - k^2}, \quad \xi^2 + \eta^2 \leq k^2 : \sqrt{k^2 - \xi^2} - \eta^2 \right\}.
\]
Vector-functions $E, H$ are the solution of the problem (1), (2) in the class of solutions outgoing into the half-space if and only if, when [5] the equality for the Fourier transforms of traces of their components on the plane $z = 0$
\[
\omega \varepsilon \gamma(\xi, \eta) e_\xi(\xi, \eta) + \xi \eta h_\xi(\xi, \eta) + (k^2 - \xi^2) h_\eta(\xi, \eta) = 0,
\]
\[
-\omega \mu \gamma(\xi, \eta) h_\xi(\xi, \eta) \xi \eta e_\xi(\xi, \eta) + (k^2 - \xi^2) e_\eta(\xi, \eta) = 0.
\]
is fulfilled.

In the case of analogues statements for the solutions of the problem outgoing into the lower half-space $z < 0$ one should change a sign of the function $\gamma(\xi, \eta)$ in formulas (3).
3. MAXWELL EQUATIONS SET IN THE ANISOTROPIC SEMI-SPACE

Consider Maxwell equations set

\[ \text{rot} \ H = i \omega \varepsilon \ E, \quad \text{rot} \ E = -i \omega \mu H \]  \hspace{1cm} (4)

in the case when dielectric constant \( \varepsilon \) is a tensor and permeance \( \mu = \mu \) is a scalar. To simplify reasoning we will consider only the case of diagonal anisotropy when

\[ \varepsilon = \varepsilon \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{pmatrix}. \]

Let, as well as before, \( k^2 = \omega^2 \mu \varepsilon \).

Let us redefine the unknown solutions of boundary value problem by zero in the lower semi-plane and consider them as distributions of slow growth at infinity. Let us pass in Equation (4) from classic derivatives to generalized ones and apply the Fourier integral transformation by all variables: \( x, y, z \rightarrow \xi, \eta, \zeta \). We will use the same denotations for Fourier transforms and for their prototypes.

If we exclude normal components \( E_z, H_z \) from obtained equations then we get for Fourier transforms of tangent components a set of four equations in the vector-matrix form

\[ \begin{pmatrix} k^2 \beta - \xi^2 \\ \eta^2 - k^2 \alpha \end{pmatrix} \begin{pmatrix} E_x \xi \eta \\ -E_y \xi \eta \end{pmatrix} + \omega \mu \zeta \begin{pmatrix} H_x \\ H_y \end{pmatrix} = i \omega \mu \begin{pmatrix} E_x \\ E_y \end{pmatrix}, \]

\[ \omega \varepsilon \zeta \begin{pmatrix} E_x \\ E_y \end{pmatrix} - \begin{pmatrix} \xi \eta \\ \eta^2 - k^2 \end{pmatrix} \begin{pmatrix} k^2 - \xi^2 \\ -\xi \eta \end{pmatrix} \begin{pmatrix} H_x \\ H_y \end{pmatrix} = i \omega \varepsilon \begin{pmatrix} E_x \\ E_y \end{pmatrix}. \]

By this the tangent components of vector \( E \) should satisfy equations

\[ k^2 \left( \begin{pmatrix} \xi^2 + \alpha \xi^2 + \eta^2 - k^2 \alpha \\ (\alpha - 1) \xi \eta \end{pmatrix} \begin{pmatrix} \beta - 1 \xi \eta \\ \zeta^2 + \xi^2 + \beta \eta^2 - k^2 \beta \end{pmatrix} \right) \begin{pmatrix} E_x \\ E_y \end{pmatrix} = i k^2 \zeta \begin{pmatrix} E_x \\ E_y \end{pmatrix} + i \omega \mu \begin{pmatrix} \xi \eta \eta^2 - k^2 \xi \eta \end{pmatrix} \begin{pmatrix} h_x \\ h_y \end{pmatrix}. \]

The determinant of coefficients matrix of this set of equations

\[ \Delta = k^4 \left\{ \zeta^4 - [(\alpha + \beta)k^2 - (1 + \alpha)\xi^2 - (1 + \beta)\eta^2] \zeta^2 + (k^2 - \xi^2 - \eta^2) (\alpha \beta k^2 - \alpha \xi^2 - \beta \eta^2) \right\} \]

can be easily decomposed into multipliers by additional supposition that anisotropy is one-axis, i.e., two of three diagonal elements of matrix are equal, either \( \alpha = 1 \), or \( \alpha = \beta \). By this two first particular cases differ from each other not essentially.

Let us consider the first variant of one-axis anisotropy.

Let \( \beta = 1 \). Denote by \( \gamma_1^2 = k^2 - \xi^2 - \eta^2 \), \( \gamma_2^2 = \alpha k^2 - \alpha \xi^2 - \eta^2 \), by this \( \Delta = k^4 (\zeta^2 - \gamma_1^2)(\zeta^2 - \gamma_2^2) \).

Assume that roots of these complex-valued expressions are extracted in such way that values \( \gamma_1 = \gamma_1(\xi, \eta) \) and \( \gamma_2 = \gamma_2(\xi, \eta) \) are either negative real numbers or pure imaginary numbers with positive imagine part.

Set of Equation (5) splits into two independent equations of the form

\[ (\zeta^2 - \gamma_1^2) \omega \varepsilon E_x = i \varphi_1(\zeta), \quad (\alpha - 1) \xi \eta \omega \varepsilon E_x + (\zeta^2 - \gamma_2^2) \omega \varepsilon E_y = i \varphi_2(\zeta). \]

The right-side parts of these equations contain the Fourier transforms of boundary functions.

By theorem Paley-Wiener solutions of Equation (6) should be analytically continued to the upper semi-plane of the complex plane by variable \( \zeta \). In addition, boundary functions should be such that after inverse Fourier transformation these solutions should contain no harmonics coming from infinity.

Consequently, distributions \( E_x, E_y, E_z, H_x, H_y, H_z \) are the solutions of the boundary value problem (4), (2) for upper semi-space in case of one-axis anisotropy of the first type and satisfy the radiation condition if and only if when conditions

\[ \omega \varepsilon \gamma_2 e_x + \xi \eta h_x + (k^2 - \xi^2) h_y = 0, \quad -\omega \mu \gamma_1 h_x + \xi \eta e_x + (k^2 - \xi^2) e_y = 0. \]

(7)
are fulfilled for Fourier transforms of boundary functions. By this the Fourier transforms of the unknown distributions

\[ E_x = \frac{i}{\zeta + \gamma_1} e_x, \quad E_y = \frac{i}{\zeta + \gamma_1} \omega \mu \gamma_1 h_x = \frac{i}{\zeta + \gamma_2} \eta h_x, \]

\[ H_x = \frac{i}{\zeta + \gamma_1} h_x, \quad H_y = -\frac{i}{\zeta + \gamma_1} \eta h_x = -\frac{i}{\zeta + \gamma_2} \omega \gamma_2 e_x, \]

\[ H_z = \frac{i}{\zeta + \gamma_1} \eta h_x + \frac{i}{\zeta + \gamma_2} \xi y, \quad H_z = -\frac{i}{\zeta + \gamma_1} \eta h_x + \frac{i}{\zeta + \gamma_2} \omega \eta e_x. \]  

(8)

For \( \alpha = \beta \) (the second variant of one-axis anisotropy) the analogous statement takes place. Distributions \( E_x, E_y, E_z, H_x, H_y, H_z \) are the solutions of the boundary value problem (4), (2) for upper semi-space in case of one-axis anisotropy of the second type and satisfy the radiation condition if and only if when conditions

\[ \gamma_1 (\eta \cdot e_x - \xi \cdot e_y) + \omega \mu (\xi \cdot h_x + \eta \cdot h_y) = 0, \quad \alpha \omega \varepsilon (\xi \cdot e_y + \eta \cdot e_x) - \gamma_2 (\eta \cdot h_x - \xi \cdot h_y) = 0, \]  

(9)

are fulfilled for Fourier transforms of boundary functions. Here \( \gamma_1^2 = k^2 - \xi^2 - \eta^2, \quad \gamma_2^2 = \alpha k^2 - \alpha \xi^2 - \eta^2. \)

It is easy to show that in the limiting case the solvability conditions (7) and (9) coincide with conditions (3).

4. REFLECTION AND REFRACTION OF WAVES ON THE MEDIA INTERFACE

Let us consider the reflection and refraction problem of electromagnetic waves coming from infinity on isotropic and anisotropic media interface. We should seek a solution of the Maxwell equations set (1) in the lower semi-space and of the Maxwell equations set (4) in the upper semi-space in class of outgoing to infinity solutions satisfying by \( z = 0 \) the conjugation conditions

\[ [z_0, E^+ - E^-](x, y) = a(x, y), \quad [z_0, H^+ - H^-](x, y) = b(x, y). \]  

(10)

Here the given vector-functions \( a(x, y) \) and \( b(x, y) \) are the differences of tangent components of electric and of magnetic wave vectors coming to plane \( z = 0 \) from below and from above. Sign + is valid for upper anisotropic semi-space of the first type and sign − is valid for lower isotropic semi-space.

Problem (1), (4), (10) is a jump problem. Its solution can be considered as a potential of special form which is convenient to use for investigating the electromagnetic wave diffraction problem on a thin conducting screen.

To determine the Fourier transforms of traces of unknown waves we have a set of linear algebraic equations consisting of the solvability conditions for the over-determined problem in the upper semi-space of the form (7)

\[ \omega \varepsilon \gamma_2^+ \cdot e_x^+ + \xi \eta \cdot h_x^+ + (k_1^2 - \xi^2) \cdot h_y^+ = 0, \quad -\omega \mu \gamma_1^+ \cdot h_x^+ + \xi \eta \cdot e_x^+ + (k_2^2 - \xi^2) \cdot e_y^+ = 0, \]

of the solvability conditions for the over-determined problem in the lower semi-space of the form (3)

\[ -\omega \varepsilon \gamma^- \cdot e_x^- + \xi \eta \cdot h_x^- + (k_1^2 - \xi^2) \cdot h_y^- = 0, \quad -\omega \mu \gamma^- \cdot h_x^- + \xi \eta \cdot e_x^- + (k_2^2 - \xi^2) \cdot e_y^- = 0, \]

and of conditions of the jump problem (10)

\[ e_x^+ - e_x^- = a_x, \quad e_y^+ - e_y^- = a_y, \quad h_x^+ - h_x^- = b_x, \quad h_y^+ - h_y^- = b_y. \]

Solution of this problem can be written down in the explicit form. To get solution of the set of Equation (4) for \( z > 0 \), it is necessary to substitute expressions of distributions \( e_x^+, e_y^+, h_x^+, h_y^+ \) into formulas (8) and to carry out the inverse Fourier transformation. To get solution of the set of Equation (1) for \( z < 0 \), it is necessary to substitute expressions \( e_x^-, e_y^-, h_x^-, h_y^- \) into analogous formulas and also to carry out the inverse Fourier transformation.
5. WAVE DIFFRACTION ON THE PLANE SCREEN

Let \( M \) be a conducting thin screen placed in the plane \( z = 0 \), and \( \mathcal{N} \) be its complement to complete plane. Let \((E_0^-, H_0^-)\) be the electromagnetic wave falling down on a screen from isotropic lower semi-space. The diffraction wave problem on a plane screen consists of the following.

It is necessary to seek solutions of set of Equation (1) for \( z < 0 \) and of set of Equation (4) for \( z > 0 \) in class of outgoing to infinity solutions satisfying for \( z = 0 \) conditions

\[
\begin{align*}
[z_0, E^+](x, y) &= 0, & \quad [z_0, E^- + E_0^-](x, y) &= 0, & \quad (x, y) &\in M, \\
[z_0, E^+-E^- - E_0^-](x, y) &= 0, & \quad [z_0, H^+ - H^- - H_0^-](x, y) &= 0, & \quad (x, y) &\in \mathcal{N}.
\end{align*}
\]

We will seek a solution \((E^\pm, H^\pm)\) of the diffraction problem in the form of a sum of two addends. First addend \((E_1^\pm, H_1^\pm)\) is a solution of the reflection and refraction wave problem on the media interface without screen, it can be found as a solution of a jump problem for \( a(x, y) = e_0^x(x, y), \quad b(x, y) = h_0^y(x, y) \).

Second addend \((E_2^\pm, H_2^\pm)\) is a perturbation from a screen. We will seek it in the form of a solution of a jump problem also. By this \( a(x, y) = 0 \) everywhere, \( b(x, y) = 0 \) on \( \mathcal{N} \). Vector-function \( b(x, y) \) on \( M \) should be found by conditions

\[
e_2^+, e_2^+ = -e_1^+, \quad e_2^-, e_2^- = -e_1^- \quad \text{on} \quad M
\]  

or by analogous conditions for traces of tangential components of vector \( E_2^- \).

The expressions of the functions \( e_2^+, e_2^- \) can be found from complete set of equations of a jump problem. Let us pass in these formulas to prototypes of distribution \( b(\xi, \eta) \). Then we obtain the set of integral equations to determine field perturbation from screen.

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REFERENCES