Stokes-Brinkman model of fluid flow through porous body of arbitrary shape

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Fully or partially porous bodies can be used as aerosol filter elements. To evaluate the capture efficiency for porous elements, the fluid flow in an ordered or random packing of porous elements must be calculated. Corresponding models approximating a circular Kuwabara cell model for a porous circular cylinder were developed by Stechkina (1979), Kirsh (2006), Mardanov et al (2016). In this work the problem formulation and results of the study of fluid flow past an arbitrary shaped porous body in a rectangular periodic cell are presented.

Let us consider the two-dimensional flow of an incompressible viscous fluid through a porous body with constant permeability $k$ in a periodic rectangular cell. The typical size $cR$ of the porous element and the velocity $U$ are selected as the length and velocity scales. The computational domain $\Omega$ consists of a homogeneous area $e\Omega$ of the external flow and a porous medium $i\Omega$ of the internal flow inside the body. Denote the periodic cell porosity by $\varepsilon = 1 - \pi / (4Rh_2)$ (fig. 1).

Figure 1. Calculation domain

The external flow in the area $\Omega^e$ is described in the Stokes flow approximation. The flow stream function $\psi^e(x, y)$ satisfies the biharmonic equation

$$\Delta^2 \psi^e = 0.$$  

On the boundaries $AE, DF$ the periodic conditions are set:

$$\psi^e(-h_2, y) = \psi^e(h_2, y), \quad \psi^e(-h_2, y) = -\psi^e(h_2, y),$$

$$\omega^e(-h_2, y) = \omega^e(h_2, y), \quad \omega^e(-h_2, y) = -\omega^e(h_2, y),$$

where $\omega^e = -\Delta \psi^e$ is the vorticity (a prime denotes differentiation with respect to the outward normal to the boundary). On the boundary $EF$: $\psi^e = h_1$, $\omega^e = 0$.

On the axis-lines $AB$ and $CD$ the symmetry conditions hold: $\psi^e = 0$, $\omega^e = 0$.

In the porous domain $\Omega^i$ the flow is described within the Brinkman model. The stream function $\psi^i(x, y)$ of the fluid flow inside the body satisfies the Brinkman equation ($S = R / \sqrt{k}$)

$$\Delta^2 \psi^i - S^2 \Delta \psi^i = 0.$$  

The symmetry conditions are taken on the line $BOC$: $\psi^i = 0$, $\omega^i = 0$. On the boundary $BC$ between the free space and the porous medium the conditions of equality of velocity vectors, pressure and tangential stresses are taken. Considering the flow stream function and vorticity these are written as:

$$\psi^e = \psi^i, \quad \omega^e = \psi^i, \quad \omega^i = S^2 \psi^i + \omega^i, \quad \omega^i = \omega^i.$$  

The boundary element method (BEM) is used to solve the above boundary value problem. Due to the form of the boundary conditions taken the method described can be used to solve the problem of fluid flow through a porous body of arbitrary shape. The method was tested for the case of a porous circular cylinder of unity radius. Comparison of the streamlines and gas capture coefficient $Q = \psi(0, 1)$ obtained by the BEM and by CFD ANSYS/FLUENT calculations are shown in figs. 2, 3, where good agreement is shown. The results of parametrical studies of porous element of various shapes will be presented at the conference.

![Classification of fluid flow streamlines at $S = 3, \varepsilon = 0.96$.](image)

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Figure 2. Fluid flow streamlines at $S = 3, \varepsilon = 0.96$.

Figure 3. The dependence $Q(S)$ for various porosity ($\varepsilon = 0.9, 0.96, 0.99$ – solid, dashed and dash-dotted lines, ANSYS/FLUENT – symbols).

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