Comparison Of Critical Rate Correlations

Firdavs A. Aliev¹, Khursheed A. Rahimov², Balabek Amzayev³, Alim F. Kemalov¹
¹Kazan Federal University, Kremlyovskayastr, 18, 420008, Kazan, Russian Fed
²Istanbul Technical University, AyazağaKampüsü 34469, Istanbul/TURKEY

Abstract-Water coning is a term used to describe the upward movement of water into the perforations of a producing well. This phenomenon can also be described as a steady and usually sharp displacement of some or all the oil production by the bottom water when the critical withdrawal rate from the well is exceeded. Water coning may lead to several serious problems. There may be loss in total recovery. Water coning is a usual problem that is faced by petroleum engineers in reservoirs having an aquifer, particularly at the bottom. The critical rate is the subject discussed mostly in the studies on water coning. This paper presents a simulation study using RUBIS, a subprogram of ECRIN. Some correlations for critical rate are analyzed and their results are compared with those from RUBIS. For reasonable comparison, parameters in the simulation program are set so that the assumptions used in correlations could be met.

Keywords: water coning, crest, stimulation, Ecrin, Rubis, Critical rate, breakthrough time

INTRODUCTION

Water coning is a term used to describe the upward movement of water into the perforations of a producing well (Ahmed, 2010). This phenomenon can also be described as a steady and usually sharp displacement of some or all the oil production by the bottom water when the critical withdrawal rate from the well is exceeded (Muscat and Wyckoff, 193). Water coning may lead to several serious problems. Moreover, there may be loss in total recovery (Ahmed, 2010). A great number of publications connected to water coning problem are appearing (Karp, 1962; Khan, 1970; Menouar, 1995; Mungan, 1979; Okwananke, 2008; Pirson, and Mehta, 1967; Smith and Pirson, 1963; Rajan and Luhning, 1993; Thomas, 2002; Wu, 1995) as it still remains essential.

Before the production, petroleum reservoirs have fluid contacts such as water-oil contact (WOC) and gas-oil contact (GOC) (Kemalov et al., 2012). As the production is initiated, these contacts change in shape and form a cone or a crest.

Critical rate is the maximum production rate which does not allow water to breakthrough into the production well. When the oil production rate becomes higher than the critical rate, WOC rises and cone becomes unstable reaching the bottom of the well (Chierici, 1995). The water cone is said to be stable if the pressure at every point on the WOC is the same as the reservoir pressure $P_{res}$ (Figure 1).

![Figure 1 – Homogeneous formation with upper portion saturated with oil and lower portion with water (Muscat and Wyckoff, 1935).](image)

Equation 1 is the required condition for water cone to stay in static condition.

$$P + \gamma_w g z = P_{res}$$ (1)

This study mainly deals with critical oil production rate and parametric analysis for the observation of some reservoir and fluid properties on the critical rate in horizontal wells. Also studies by some authors on water coning in horizontal wells and correlations for critical production rate and breakthrough time calculations are mentioned. In addition, one example problem is solved using RUBIS (Ecrin v4.20, 2013) and compared with the results from some correlations.

METHODS

Breakthrough time is the time when water from aquifer reaches the production well. One of the primary factors leading to coning is pressure drawdown. There is a substantial pressure drawdown near the wellbore displayed by a vertical well (Makinde et al., 2011). Muscat and Wyckoff (1935) point out the first reason in pressure drop between the reservoir boundary and the
points below the bottom of the well is greater than the hydrostatic head of the given water column. Another reason is related to viscous and gravity forces. The latter is associated with density difference between the oil and water. When the dynamic, or viscous, forces exceed the static forces, this brings about coning. The forces that have an effect on water coning are capillary, viscous, and gravity forces. Gravity forces act in vertical direction and cause the fluid to rise due to density difference. At any time there is an equilibrium between viscous and gravity forces. Once this balance is destroyed, more specifically, when the viscous forces exceed the gravitational ones, cone will break into the well. However, if the opposite circumstance is the case, the cone will not move backward, and therefore, it is called a stable cone. On the other hand, if the pressure in the system is in unsteady state, the cone, which is now known as an unstable, will proceed towards the well until the steady-state condition is reached. The reason for water cone to become unstable is that upward dynamic force is extremely high and is not possible to balance with the weight of water below.

Empirical correlations have been developed to estimate the critical production rate in vertical wells in the literature (Permadi and Jayadi, 2010). Some of them are discussed below.

**Hoyland-Papatzacos-Skjaeveland Correlation**

Hoyland, Papatzacos, and Skjaeveland (Hoyland et al., 1989; Papatzacos et al., 1989) suggested analytical and numerical correlations for prediction of critical oil rate. They assume bottom water coning in anisotropic, homogeneous systems where the well is completed from the top of the formation.

![Figure 2](image)

Figure 2 – Partially penetrating well with boundary conditions (Hoyland et al. 1989)

The shape of the cone is neglected in this approach. In the procedure for calculation, first dimensionless radius is calculated from Equation 2. Next, dimensional critical rate for different fractional penetrations is determined. Then the dimensionless critical rate, \( q_{CD} \) as a function of well penetration is plotted as in Figure 3. Fractional well penetration \( \left( \frac{h_w}{h} \right) \) is found and plot is extrapolated to find the dimensionless critical rate. Finally, using the Equation 3 critical rate is calculated.

\[
q_{oc} = 0.246 \times 10^{-4} \left[ \frac{h^2 (\rho_o - \rho_w) k_h}{\mu_o B_o} \right] q_{CD}
\]

For the isotropic reservoir, where \( k_v = k_h \), the relationship developed is given as in Equation 4.

\[
q_{oc} = 0.924 \times 10^{-4} \frac{k_v (\rho_o - \rho_w)}{\mu_o B_o} \left[ \left( 1 - \frac{h_w}{h} \right)^{\frac{1}{3}} \times h^{\frac{1}{3}} \times h^{\frac{1}{3}} \times h^{\frac{1}{3}} \right] \quad (4)
\]

**Chaperon Correlation**
Chaperon (1986) assumes anisotropic formation. It is also assumed that the completion interval is too short. Chaperon’s relationship accounts for the distance between the production well and the boundary. The relation is given in the following equation.

\[
q_{oc} = 0.0783 \times 10^{-4} \frac{k_b (h - h_w)}{\mu_a B_o} \rho_o (\rho_o - \rho_a) q_e^* \tag{5}
\]

\[
q_e^* = 0.7311 + (1.943/\alpha^*)
\]

\[
\alpha^* = \frac{r_e}{h} \sqrt{k_b}
\]

**Guo-Lee Correlation**

Guo and Lee (1992) assume a partially penetrating well in an isotropic formation. The relation is shown in Equation 6.

\[
q_{oc} = \frac{7.08 \times 10^{-3} k_b \Delta P}{\mu_a} \left[ \frac{h_w}{h} - 0.021 \left( \frac{h_w}{h} \right)^2 \right] \left[ \frac{h_w}{h} \right] \left[ \frac{h_w}{h} \right] \left[ \frac{h_w}{h} \right]
\]

\[
q_{oc} = \frac{k_b h^2 \Delta P}{325.7 \mu_a B_o} \left[ 0.546 - 0.021 \left( \frac{h_w}{h} \right) - 0.525 \left( \frac{h_w}{h} \right)^2 \right]
\]

\[
(7)
\]

Three correlations for critical rate calculation in horizontal wells are also discussed. These are Chaperon (1986), Ozkan and Raghavan (1990), and Giger (1989) correlations.

**Chaperon Correlation for Critical Rate**

In isotropic formations with steady-state or pseudo steady-state flow conditions Chaperon (1986) proposes that horizontal wells allow higher critical rates than vertical wells. The equation derived assumes the well to be located at the bottom of oil zone. Initially she observed the effect of forces on a stable crest. It is determined that balance between viscous and gravity forces keep the crest stable. The equation derived for the flow potential is expressed in Equation 14.

\[
\varphi(x, z_e) = q \frac{\mu_o}{2\pi Lk} \log \left( \frac{\pi x}{h} - \cos \frac{\pi z_e}{h} \right)
\]

Finally, the author equated the viscous potential difference to the gravity potential difference and proposed equation predicting critical rate as in Equation 15.

\[
q_{oc} = 0.0783 \times 10^{-4} \frac{L q_e^*}{y_c} \rho_o (\rho_o - \rho_a) \frac{k_b (D_h)^2}{\mu_o B_o}
\]

\[
(15)
\]

\[
q_e^* = 3.9624955 + 0.0616438 \alpha^* - 0.000504 (\alpha^*)^2
\]

The equation is constrained by \( 1 \leq \alpha^* < 70 \), and \( 2 y_c < 4L \) where

\[
4L = \frac{y_c}{h} \sqrt{k_v}
\]

**Ozkan-Raghavan Correlation**

Ozkan and Raghavan (1990) assumed an infinitely large reservoir and sufficiently long well. The well is placed at the top of oil zone. The authors proposed Equation 16 below to find the critical rate of horizontal wells.

\[
q_{oc} = \frac{k_b h^2 \Delta P}{325.7 \mu_a B_o} \left[ 1.0194 - 0.1021 \left( \frac{h_w}{h} \right) - 0.2807 \left( \frac{h_w}{h} \right)^2 \right] k_o
\]

where

\[
L_D = \frac{L}{2h} \sqrt{k_v}
\]

**Giger Correlation**

For derivation the critical rate correlation Giger (1989) located the well near the top of oil zone. In addition, he assumes the well extends throughout the oil zone. The external boundary of reservoir is closed to lateral flow (Figure 4). For such a case Giger (1989) proposed Equation 17 for calculating critical rate.

\[
q_{oc} = \frac{3 \times 10^{-3} k_b \Delta P}{\mu_a B_o} \left[ (1 + 16y^2)^{0.5} \right] - 1
\]

\[
(17)
\]

![Figure 4](image-url) - Bottom water-drive case during production (Giger, 1989).

**ANALYSIS AND RESULTS**

In the present study, an example problem was solved by means of these correlations and RUBIS (Ecrinv4.20., 2013). Approaches used for the vertical well were those developed by Chaperon (1986), Ozkan and Raghavan (1990), Guo and Lee (1992), and Hoyland et al. (1989). For horizontal well, correlations used for comparison were Chaperon (1986), Özkan and
Raghavan (1990), and Giger (1989) correlations. The
field data for the example problem for both vertical and
horizontal wells are the same and shown in Table 1.

Table 1 – Field data for the example problem.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservoir temperature</td>
<td>212 °F</td>
</tr>
<tr>
<td>Reservoir initial Pressure</td>
<td>5000 psia</td>
</tr>
<tr>
<td>API gravity</td>
<td>32.8</td>
</tr>
<tr>
<td>Vertical depth</td>
<td>6000 ft</td>
</tr>
<tr>
<td>Reservoir oil thickness</td>
<td>42 ft</td>
</tr>
<tr>
<td>Reservoir drainage radius</td>
<td>1053 ft</td>
</tr>
<tr>
<td>Wellbore radius</td>
<td>0.29 ft</td>
</tr>
<tr>
<td>Vertical anisotropy ratio</td>
<td>0.1</td>
</tr>
<tr>
<td>Horizontal well length</td>
<td>660 ft</td>
</tr>
<tr>
<td>Reservoir drainage radius</td>
<td>1053 ft</td>
</tr>
<tr>
<td>Wellbore radius</td>
<td>0.29 ft</td>
</tr>
<tr>
<td>Vertical anisotropy ratio</td>
<td>0.1</td>
</tr>
<tr>
<td>Horizontal permeability</td>
<td>37 md</td>
</tr>
<tr>
<td>Vertical permeability</td>
<td>3.7 md</td>
</tr>
<tr>
<td>Mobility ratio</td>
<td>3.27</td>
</tr>
<tr>
<td>Oil viscosity</td>
<td>1.44 cp</td>
</tr>
</tbody>
</table>

Several cases were run considering both anisotropic
and isotropic formations. RUBIS was run according to
the assumptions in each correlations in order to make a
reasonable comparison. The results are summarized in
Table 2, for vertical and horizontal wells, respectively.

Table 2 – Critical rate calculation approaches for vertical and horizontal wells

<table>
<thead>
<tr>
<th>Approach</th>
<th>Vertical Anisotropy</th>
<th>Critical Rate, STB/day</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chaperon RUBIS</td>
<td></td>
<td>5.4 5.7</td>
</tr>
<tr>
<td>Guo-Lee Hoyland et al.</td>
<td>( \frac{k_v}{k_h} = 0.1 )</td>
<td>1.53 2.2 2.62</td>
</tr>
<tr>
<td>Ozkan-Raghavan RUBIS</td>
<td></td>
<td>10.73 10.70</td>
</tr>
<tr>
<td>Hoyland et al. RUBIS</td>
<td>( \frac{k_v}{k_h} = 1 )</td>
<td>1.69 2.39</td>
</tr>
</tbody>
</table>

DISCUSSION

As seen from Table 2, results from Guo and Lee (1992) and Hoyland et al. (1989) correlations are comparable to each other. The ratio of completed interval to reservoir thickness is 0.571 in their case. Chaperon (1986) correlation yields higher critical rate since it is assumed that the completion interval of the well is very short. Ozkan and Raghavan (1990) correlation proved to yield even higher result for critical rate. The completion interval is the same as for Guo and Lee (1992) and Hoyland et al. (1989) correlations. However, such a high value for critical rate may be the result of assuming an infinitely large reservoir.

As seen from the results in Table 2, critical rate values obtained by Ozkan and Raghavan (1990) correlation and RUBIS proved to be close to each other. The horizontal well was positioned at the top of the reservoir and the reservoir is assumed as infinitely large. Therefore, the critical rate in this case is the highest. Similarly, Giger (1989) assumes the horizontal well at the top of the reservoir; however, the drainage
radius was used as given in the problem, that is, 1053 ft. That is why, the critical rate in this case is lower.

Chaperon (1986) considers the horizontal well position at about the one third of the reservoir oil thickness. Although the well is placed at the top for Giger (1989) correlation, his critical rate is expected to be higher than the one obtained from Chaperon (1986) correlation.

SUMMARY
Water coning is one of the severe problems encountered in petroleum engineering. Therefore, a great importance should be given to the studies on this phenomenon. Many reservoirs are bottom water drive, and oil from these reservoirs is usually produced at higher rates than the critical rate. This generally results in early breakthrough of water from aquifer into the producing well.

CONCLUSION
In present paper we made an overview of the existing literature data on the terms of water coning problem. It was revealed, that there is a plenty of correlations done to determine the optimal rate of oil producing to avoid water coning. Many correlations are presented to find the critical rate, but none of them are common and exact. Each correlation has its specific assumptions which makes it applicable for the certain reservoir.

CONFLICT OF INTEREST
The author confirms that the data do not contain any conflict of interest.

ACKNOWLEDGEMENTS
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REFERENCES


NOMENCLATURE

\( B_o \) : oil formation volume factor, bbl/STB

\( D \) : lateral length of oil zone, ft

\( Db \) : distance between the WOC and the horizontal well, ft

\( E_s \) : sweep efficiency, dimensionless

\( f_d \) : dimensionless parameter

\( g \) : acceleration of gravity, ft/s^2

\( \Delta g \) : difference between pressure gradients of water and oil, psi/ft

\( h \) : oil zone thickness, ft

\( h_L \) : location of the horizontal well

\( h_p \) : perforated interval, ft

\( h_w \) : completed interval of the vertical well, ft

\( k \) : permeability to oil, md

\( k_h \) : horizontal permeability, md

\( k_o \) : oil permeability, md

\( (k_{to})_{swc} \) : oil relative permeability at connate water saturation

\( (k_{tw})_{hor} \) : water relative permeability at residual oil saturation

\( k_v \) : vertical permeability, md

\( L \) : horizontal well length, ft

\( L_D \) : dimensionless horizontal well length

\( M \) : water-oil mobility ratio

\( p_p \) : hydrostatic pressure in the water zone, psia

\( p_{res} \) : reservoir pressure, psia

\( p_{wf} \) : bottomhole flowing pressure, psia

\( q \) : actual oil rate, m^3/hour

\( q_{CD} \) : dimensionless critical flow rate

\( q_D \) : dimensionless rate

\( q_o \) : oil flow rate, STB/day

\( q_e^* \) : dimensionless function of Joshi (Joshi, 1988; Joshi, 1991) which is a function of \( \alpha^* \)

\( Q_o \) : cumulative oil produced, MMSTB

\( Q_{oc} \) : critical oil rate, STB/day

\( Q_w \) : cumulative water produced, MMSTB

\( r_D \) : dimensionless radius

\( r_e \) : radius of the reservoir, ft

\( S_{or} \) : residual oil saturation, fraction

\( S_{swc} \) : connate water saturation, fraction

\( t \) : thickness of oil horizon, ft

\( t_{BT} \) : time to breakthrough, days

\( (t_D)_{BT} \) : dimensionless breakthrough time

\( x \) : aside from the horizontal well

\( x_A \) : location of a constant pressure boundary, ft

\( y \) : vertical distance between initial WOC and horizontal well, ft

\( y_e \) : half distance between two lines of horizontal wells, ft

\( Z \) : dimensionless cone height

\( z \) : height of the water cone, ft

\( z_k \) : coordinate along vertical axis

\( Z_s \) : well to cone apex distance, ft

Greek letters:

\( \alpha \) : constant for Sobocinski-Cornelius correlation (Sobocinski and Cornelius, 1964)

\( \alpha^* \) : constant in Chaperon correlation

\( \gamma_w \) : water specific gravity

\( \mu_o \) : oil viscosity, cp

\( \rho_o \) : oil density, lb/ft^3

\( \rho_w \) : water density, lb/ft^3

\( \phi \) : porosity, fraction

\( \phi \) : flow potential or gravity potential