Inflationary models in the Einstein and modified gravity producing the best fit to present observational data

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International Winter School on Gravitation,
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"Petrov Readings – 2014"

Kazan Federal University, Kazan, 17 - 19.02.2014

Inflationary spectral predictions and observations

$f(R)$ gravity and $R + R^2$ inflationary model

Relation to Higgs inflation in scalar-tensor gravity

Generality of inflation in the most favoured models

Conclusions

Four epochs of the history of the Universe

$H \equiv \frac{\dot{a}}{a}$ where $a(t)$ is a scale factor of an isotropic homogeneous spatially flat universe (a Friedmann-Lemaitre-Robertson-Walker background):

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2)$$

The history of the Universe in one line according to the present paradigm:

$$? \longrightarrow DS \Longrightarrow FLRWRD \Longrightarrow FLRWMD \Longrightarrow \overline{DS} \longrightarrow ?$$

$$|\dot{H}| << H^2 \Longrightarrow H = \frac{1}{2t} \Longrightarrow H = \frac{2}{3t} \Longrightarrow |\dot{H}| << H^2$$

$$p \approx -\rho \Longrightarrow p = \rho/3 \Longrightarrow p \ll \rho \Longrightarrow p \approx -\rho$$

FLRW dynamics with a scalar field

In the absence of spatial curvature and other matter:

$$H^2 = \frac{\kappa^2}{3}\left(\frac{\dot{\phi}^2}{2} + V(\phi)\right)$$

$$\dot{H} = -\frac{\kappa^2}{2}\dot{\phi}^2$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

where $\kappa^2 = 8\pi G$ ($\hbar = c = 1$).

Reduction to the first order equation

It can be reduced to the first order Hamilton-Jacobi-like equation for $H(\phi)$:

$$\frac{2}{3\kappa^2}\left(\frac{dH}{d\phi}\right)^2 = H^2 - \frac{\kappa^2}{3}V(\phi)$$

Time dependence is determined using the relation

$$t = -\frac{\kappa^2}{2}\int\left(\frac{dH}{d\phi}\right)^{-1} d\phi$$

However, during oscillations of ϕ, $H(\phi)$ acquires non-analytic behaviour of the type $const + \mathcal{O}(|\phi - \phi_1|^{3/2})$ at the points where $\dot{\phi} = 0$, and then the correct matching with another solution is needed.

Inflationary slow-roll dynamics

Slow-roll occurs if: $|\ddot{\phi}| \ll H|\dot{\phi}|$, $\dot{\phi}^2 \ll V$, and then $|\dot{H}| \ll H^2$.

Necessary conditions: $|V'| \ll \kappa V$, $|V''| \ll \kappa^2 V$. Then

$$H^2 \approx \frac{\kappa^2 V}{3}, \quad \dot{\phi} \approx -\frac{V'}{3H}, \quad N \equiv \ln \frac{a_f}{a} \approx \kappa^2 \int_{\phi_f}^{\phi} \frac{V}{V'} \, d\phi$$

First obtained in A.A. Starobinsky, Sov. Astron. Lett. 4, 82 (1978) in the $V = \frac{m^2\phi^2}{2}$ case and for a bouncing model.

General scheme of generation of perturbations

A genuine quantum-gravitational effect: a particular case of the effect of particle-antiparticle creation by an external gravitational field. Requires quantization of a space-time metric. Similar to electron-positron creation by an electric field. From the diagrammatic point of view: an imaginary part of a one-loop correction to the propagator of a gravitational field from all quantum matter fields including the gravitational field itself, too.

One spatial Fourier mode $\propto e^{i\mathbf{kr}}$ is considered.

For scales of astronomical and cosmological interest, the effect occurs at the primordial de Sitter (inflationary) stage when $k \sim a(t)H(t)$ where $k \equiv |\mathbf{k}|$ (the first Hubble radius crossing).

After that, for a very long period when $k \ll aH$ until the second Hubble radius crossing (which occurs rather recently at the FLRWRD or FLRWMD stages), there exist one mode of scalar (adiabatic, density) perturbations and two modes of tensor perturbations (primordial gravitational waves) for which metric perturbations are constant (in some gauge) and independent of (unknown) local microphysics due to the causality principle.
In this regime in the coordinate representation:

$$ds^2 = dt^2 - a^2(t)(\delta_{lm} + h_{lm})dx^l dk^m, \quad l, m = 1, 2, 3$$

$$h_{lm} = 2\zeta(\mathbf{r})\delta_{lm} + g(\mathbf{r})e_{lm}, \quad e_l^l = 0, \; g_{,l}e_m^l = 0, \; e_{lm}e^{lm} = 1$$

Classical-to-quantum transition

Quantum-to-classical transition: in fact, metric perturbations h_{lm} are quantum (operators in the Heisenberg representation) and remain quantum up to the present time. But, after omitting of a very small part, decaying with time, they become commuting and, thus, equivalent to classical (c-number) stochastic quantities with the Gaussian statistics (up to small terms quadratic in ζ, g).

Spectral predictions of the one-field inflationary scenario in GR

Scalar (adiabatic) perturbations:

$$P_\zeta(k) = \frac{H_k^4}{4\pi^2\dot{\phi}^2} = \frac{GH_k^4}{\pi|\dot{H}|_k} = \frac{128\pi G^3 V_k^3}{3V_k'^2}$$

where the index k means that the quantity is taken at the moment $t = t_k$ of the Hubble radius crossing during inflation for each spatial Fourier mode $k = a(t_k)H(t_k)$. Through this relation, the number of e-folds from the end of inflation back in time $N(t)$ transforms to $N(k) = \ln\frac{k_f}{k}$ where $k_f = a(t_f)H(t_f)$, t_f denotes the end of inflation.
The spectral slope

$$n_s(k) - 1 \equiv \frac{d\ln P_\zeta(k)}{d\ln k} = \frac{1}{\kappa^2}\left(2\frac{V_k''}{V_k} - 3\left(\frac{V_k'}{V_k}\right)^2\right)$$

Tensor perturbations - primordial gravitational waves (A.A. Starobinsky, JETP Lett. 50, 844 (1979)):

$$P_g(k) = \frac{16 G H_k^2}{\pi}; \quad n_g(k) \equiv \frac{d \ln P_g(k)}{d \ln k} = -\frac{1}{\kappa^2}\left(\frac{V_k'}{V_k}\right)^2$$

The consistency relation:

$$r(k) \equiv \frac{P_g}{P_\zeta} = \frac{16|\dot{H}_k|}{H_k^2} = 8|n_g(k)|$$

Tensor perturbations are always suppressed by at least the factor $\sim 8/N(k)$ compared to scalar ones. For the present Hubble scale, $N(k_H) = (50 - 60)$.

Potential reconstruction from scalar power spectrum

In the slow-roll approximation:

$$\frac{V^3}{V'^2} = CP_\zeta(k(t(\phi))), \quad C = const$$

Changing variables for ϕ to $N(\phi)$ and integrating, we get:

$$\frac{1}{V(N)} = -\frac{\kappa^2}{C}\int \frac{dN}{P_\zeta(N)}$$

$$\kappa\phi = \int dN \sqrt{\frac{d \ln V(N)}{dN}}$$

Some ambiguity in the form of $V(\phi)$ because of an integration constant in the first equation. Information about $P_g(k)$ helps to remove this ambiguity.

In particular, if primordial GW are not discovered in the order $n_s - 1$:

$$r \ll 8|n_s - 1| \approx 0.3 \ ,$$

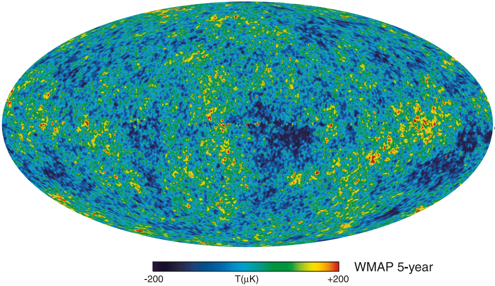
then $\left(\frac{V'}{V}\right)^2 \ll \left|\frac{V''}{V}\right|, \ |n_g| = \frac{r}{8} \ll |n_s - 1|, \ |n_g|N \ll 1 \ .$

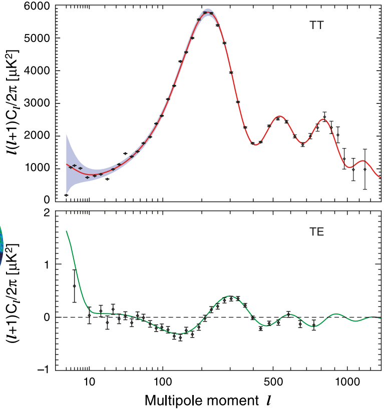
This is possible only if $V = V_0 + \delta V, \ |\delta V| \ll V_0$ – a plateau-like potential. Then

$$\delta V(N) = \frac{\kappa^2 V_0^2}{C} \int \frac{dN}{P_\zeta(N)}$$

$$\kappa\phi = \int \frac{dN}{\sqrt{V_0}} \sqrt{\frac{d(\delta V(N))}{dN}}$$

Here, integration constants renormalize V_0 and shift ϕ. Thus, the unambiguous determination of the form of $V(\phi)$ without knowledge of $P_g(k)$ becomes possible.


-200
T(μK)
+200
WMAP 5-year


TT
TE
$l(l+1)C_l/2\pi$ [μK^2]
$(l+1)C_l/2\pi$ [μK^2]
6000
5000
4000
3000
2000
1000
0
2
1
0
–1
10
100
500
1000
Multipole moment l

Combined results from Planck and other experiments

P. A. R. Ade et al., arXiv:1303.5082

Model	Parameter	*Planck*+WP	*Planck*+WP+lensing	*Planck* + WP+high-ℓ	*Planck*+WP+BAO
ΛCDM + tensor	n_s	0.9624 ± 0.0075	0.9653 ± 0.0069	0.9600 ± 0.0071	$0.9643 + 0.0059$
	$r_{0.002}$	< 0.12	< 0.13	< 0.11	< 0.12
	$-2\Delta \ln \mathcal{L}_{max}$	0	0	0	-0.31

Table 4. Constraints on the primordial perturbation parameters in the ΛCDM+r model from *Planck* combined with other data sets. The constraints are given at the pivot scale $k_* = 0.002$ Mpc^{-1}.

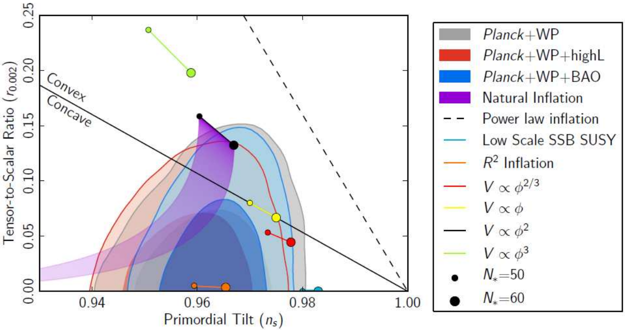


Fig. 1. Marginalized joint 68% and 95% CL regions for n_s and $r_{0.002}$ from *Planck* in combination with other data sets compared to the theoretical predictions of selected inflationary models.

Remaining models

I. Disfavoured at 95% and more CL.

1. Scale-free (or, the Harrison-Zeldovich) spectrum $n_s = 1$.
2. Power-law inflation (exponential $V(\phi)$).
3. Power-law $V(\phi) \propto \phi^n$ with $n \geq 2$.

II. Lying between 68% and 95% CL.

1. Other monomial potentials.
2. New inflation (or, the hill-top model with $V(\phi) = V_0 - \frac{\lambda\phi^4}{4}$).
3. Natural inflation.

III. Most favoured: models with $n_s - 1 = \frac{2}{N} \approx 0.04$ and $r \ll 8|n_s - 1|$.

1. $R + R^2$ model (AS, 1980).
2. A scalar field model with $V(\phi) = \frac{\lambda\phi^4}{4}$ at large ϕ and strong non-minimal coupling to gravity $\xi R\phi^2$ with $\xi < 0,\ |\xi| \gg 1$, including the Higgs inflationary model.
3. Minimally coupled (GR) models with a very flat $V(\phi)$: if $n_s(k) - 1 = \frac{2}{N(k)}$ (i.e. $P_\zeta(N) \propto N^2$) and $r \ll 8|n_s - 1|$ for all $N(k)$, then:

$$V(\phi) = V_0 \left(1 - \exp(-\alpha\kappa\phi)\right)$$

with $\alpha\kappa\phi \gg 1$ but α not very small.

All these models have $r \sim 10/N^2$, namely $r = \frac{12}{N^2} \approx 0.4\%$ for the models 1 and 2, and $r = \frac{8}{\alpha^2 N^2}$ for the third model.

$f(R)$ gravity

The simplest model of modified gravity (= geometrical dark energy) considered as a phenomenological macroscopic theory in the fully non-linear regime and non-perturbative regime.

$$S = \frac{1}{16\pi G} \int f(R)\sqrt{-g}\, d^4x + S_m$$

$$f(R) = R + F(R), \quad R \equiv R^\mu_\mu \ .$$

One-loop corrections depending on R only (not on its derivatives) are assumed to be included into $f(R)$. The normalization point: at laboratory values of R where the scalaron mass (see below) $m_s \approx \mathrm{const}$.

Field equations

$$\frac{1}{8\pi G}\left(R_\mu^\nu - \frac{1}{2}\,\delta_\mu^\nu R\right) = -\left(T_{\mu\,(vis)}^\nu + T_{\mu\,(DM)}^\nu + T_{\mu\,(DE)}^\nu\right) ,$$

where $G = G_0 = const$ is the Newton gravitational constant measured in laboratory and the effective energy-momentum tensor of DE is

$$8\pi G T_{\mu\,(DE)}^\nu = F'(R)\,R_\mu^\nu - \frac{1}{2}\,F(R)\delta_\mu^\nu + \left(\nabla_\mu \nabla^\nu - \delta_\mu^\nu \nabla_\gamma \nabla^\gamma\right) F'(R) .$$

Because of the need to describe DE, de Sitter solutions in the absence of matter are of special interest. They are given by the roots $R = R_{dS}$ of the algebraic equation

$$Rf'(R) = 2f(R) .$$

In the special case $f(R) \propto R^2$, the de Sitter space-time with *any* curvature is a solution.

Degrees of freedom

I. In quantum language: particle content.
1. Graviton – spin 2, massless, transverse traceless.
2. Scalaron – spin 0, massive, mass - R-dependent:
$m_s^2(R) = \frac{1}{3f''(R)}$ in the WKB-regime.

II. Equivalently, in classical language: number of free functions of spatial coordinates at an initial Cauchy hypersurface.
Six, instead of four for GR – two additional functions describe massive scalar waves.

Thus, $f(R)$ gravity is a non-perturbative generalization of GR. It is equivalent to scalar-tensor gravity with $\omega_{BD} = 0$ (if $f''(R) \neq 0$).

Transformation to the Einstein frame and back

In the Einstein frame, free particles of usual matter do not follow geodesics and atomic clocks do not measure proper time.

From the Jordan (physical) frame to the Einstein one:

$$g^{E}_{\mu\nu} = f' g^{J}_{\mu\nu}, \quad \kappa\phi = \sqrt{\frac{3}{2}} \ln f', \quad V(\phi) = \frac{f'R - f}{2\kappa^2 f'^2}$$

Inverse transformation:

$$R = \left(\sqrt{6}\kappa \frac{dV(\phi)}{d\phi} + 4\kappa^2 V(\phi) \right) \exp\left(\sqrt{\frac{2}{3}}\kappa\phi \right)$$

$$f(R) = \left(\sqrt{6}\kappa \frac{dV(\phi)}{d\phi} + 2\kappa^2 V(\phi) \right) \exp\left(2\sqrt{\frac{2}{3}}\kappa\phi \right)$$

$V(\phi)$ should be at least C^1.

Background FRW equations in $f(R)$ gravity

$$ds^2 = dt^2 - a^2(t)\left(dx^2 + dy^2 + dz^2\right)$$

$$H \equiv \frac{\dot{a}}{a}\,, \quad R = 6(\dot{H} + 2H^2)$$

The trace equation (4th order)

$$\frac{3}{a^3}\frac{d}{dt}\left(a^3\frac{df'(R)}{dt}\right) - Rf'(R) + 2f(R) = 8\pi G(\rho_m - 3p_m)$$

The 0-0 equation (3d order)

$$3H\frac{df'(R)}{dt} - 3(\dot{H} + H^2)f'(R) + \frac{f(R)}{2} = 8\pi G\rho_m$$

Reduction to the first order equation

In the absence of spatial curvature and $\rho_m = 0$, it is always possible to reduce these equations to a first order one using the transformation to the Einstein frame and the Hamilton-Jacobi-like equation for a minimally coupled scalar field in a spatially flat FLRW metric:

$$\frac{2}{3\kappa^2}\left(\frac{dH_E(\phi)}{d\phi}\right)^2 = H_E^2 - \frac{\kappa^2}{3}V(\phi)$$

where

$$H_E \equiv \frac{d}{dt_E}\ln a_E = \frac{1}{\sqrt{f'}}\left(\ln a + \frac{1}{2}\ln f'\right)$$

$$= \frac{1}{2\sqrt{f'}}\left(3H + \frac{\dot{H}}{H} - \frac{f}{6Hf'}\right)$$

From a solution $H_E(\phi(R))$ of this equation, the scale factor $a(t)$ follows in the parametric form:

$$\ln a = -\frac{1}{2}\ln f'(R) - \frac{3}{4}\int \left(\frac{f''}{f'}\right)^2 H_E(R) \left(\frac{dH_E(R)}{dR}\right)^{-1} dR$$

$$t = -\frac{3}{4}\int \left(\frac{f''}{f'}\right)^2 \left(\frac{dH_E(R)}{dR}\right)^{-1} dR$$

Most favoured inflationary models in $f(R)$ gravity

The simplest one (Starobinsky, 1980):

$$f(R) = R + \frac{R^2}{6M^2}$$

with small one-loop quantum gravitational corrections producing the scalaron decay via the effect of particle-antiparticle creation by gravitational field (so all present matter is created in this way).
During inflation ($H \gg M$): $H = \frac{M^2}{6}(t_f - t), \quad |\dot{H}| \ll H^2$.

The only parameter M is fixed by observations – by the primordial amplitude of adiabatic (density) perturbations in the gravitationally clustered matter component:

$$M = 2.9 \times 10^{-6} M_{Pl} \ (50/N) \ ,$$

where $N \sim (50 - 55)$, $M_{Pl} = \sqrt{G} \approx 10^{19}$ GeV.

Post-inflationary evolution

First order equation:

$$x = H^{3/2}, \;\; y = \frac{1}{2}H^{-1/2}\dot{H}, \;\; dt = \frac{dx}{3x^{2/3}y}$$

$$\frac{dy}{dx} = -\frac{M^2}{12x^{1/3}y} - 1$$

The y-axis corresponds to inflection points $\dot{a} = \ddot{a} = 0, \dddot{a} \neq 0$.
A curve reaching the y-axis at the point $(0, y_0 < 0)$ continues from the point $(0, -y_0)$ to the right.
Late-time asymptotic:

$$a(t) \propto t^{2/3}\left(1 + \frac{2}{3Mt}\sin M(t - t_1)\right), \;\; R \approx -\frac{8M}{3t}\sin M(t - t_1)$$

$$< R^2 > = \frac{32M^2}{9t^2}, \;\; 8\pi G\rho_{s,eff} = \frac{3 < R^2 >}{8M^2} = \frac{4}{3t^2} \propto a^{-3}$$

Scalaron decay and creation of matter

Transition to the FLRWRD stage: occurs through the same mechanism which has been used for generation of perturbations: creation of particle-antiparticle pairs of all quantum matter fields by fast oscillations of R. Technically: one-loop quantum corrections from all matter quantum fields have to be added to the action of the $R + R^2$ gravity. In the particle interpretation: scalaron decays into particles and particles with the energy $E = M/2$.

The most effective decay channel: into minimally coupled scalars with $m \ll M$. Then the formula obtained in Ya. B. Zeldovich and A. A. Starobinsky, JETP Lett. 26, 252 (1977) can be used:

$$\frac{1}{\sqrt{-g}}\frac{d}{dt}(\sqrt{-g}\,n_s) = \frac{R^2}{576\pi}$$

The corresponding (partial) decay rate is $\Gamma = \frac{GM^3}{24} \sim 10^{24}\ \mathrm{s}^{-1}$, that leads to the maximal temperature $T \approx 3 \times 10^9$ GeV at the beginning of the FLRWRD stage and to $N \approx 53$ for the reference scale in the CMB measurements ($k/a(t_0) = 0.05\ \mathrm{Mpc}^{-1}$), see D. S. Gorbunov, A. G. Panin, Phys. Lett. B 700, 157 (2011) and F. Bezrukov, D. Gorbunov, Phys. Lett. B 713, 365 (2012) for more details.

Predictions for primordial perturbation spectra:

$$n_s = 1 - \frac{2}{N} \approx 0.96$$

$$r = \frac{12}{N^2} \approx 0.004$$

.

Generic $f(R)$ inflationary model with
$n_s(k) = 1 - \frac{2}{N(k)}$; $r(k) \sim \frac{10}{N^2(k)} \ll 1 - n_s$ has
$V(\phi) = V_0 \left(1 - \exp(-\alpha\kappa\phi)\right)$ in the Einstein frame.
In the Jordan (physical) frame, this means that

$$f(R) = \frac{R^2}{6M^2} + CR^{2-\alpha\sqrt{3/2}}$$

for large R.
Less natural, has one more free parameter, but still possible.

One viable microphysical model leading to such form of $f(R)$

A non-minimally coupled scalar field with a large negative coupling ξ (for this choice of signs, $\xi_{conf} = \frac{1}{6}$):

$$L = \frac{R}{16\pi G} - \frac{\xi R \phi^2}{2} + \frac{1}{2}\phi_{,\mu}\phi^{,\mu} - V(\phi), \quad \xi < 0, \quad |\xi| \gg 1 \; .$$

Leads to $f' > 1$.
Recent development: the Higgs inflationary model (F. Bezrukov and M. Shaposhnikov, 2008). In the limit $|\xi| \gg 1$, the Higgs scalar tree level potential $V(\phi) = \frac{\lambda(\phi^2 - \phi_0^2)^2}{4}$ just produces $f(R) = \frac{1}{16\pi G}\left(R + \frac{R^2}{6M^2}\right)$ with $M^2 = \lambda/24\pi\xi^2 G$ and $\phi^2 = |\xi| R/\lambda$ (for this model, $|\xi| G\phi_0^2 \ll 1$).

SM loop corrections to the tree potential leads to $\lambda = \lambda(\phi)$, then the same expression for $f(R)$ follows with

$$M^2 = \frac{\lambda(\phi(R))}{24\pi\xi^2 G}\left(1 + \mathcal{O}\left(\frac{d\ln\lambda(\phi(R))}{d\ln\phi}\right)^2\right).$$

.

The approximate shift invariance $\phi \to \phi + c, \;\; c = \mathrm{const}$ permitting slow-roll inflation for a minimally coupled inflaton scalar field transforms here to the approximate scale (dilatation) invariance
$\phi \to c\phi, \;\; R \to c^2 R, \;\; x^\mu \to x^\mu/c, \;\; \mu = 0, ..3$
for curvatures exceeding that at the end of inflation in the physical (Jordan) frame.

$$S = \frac{N^2}{288\pi^2\Delta_\xi^2}\int R^2\sqrt{-g}\,d^4x \approx 4\times 10^8\int R^2\sqrt{-g}\,d^4x$$

Of course, this symmetry needs not be fundamental, i.e. existing in some more microscopic model at the level of its action.

Generality of inflation

Theorem. In these models, there exists an open set of classical solutions with a non-zero measure in the space of initial conditions at curvatures much exceeding those during inflation which have a metastable inflationary stage with a given number of e-folds.

For the GR inflationary model this follows from the generic late-time asymptotic solution for GR with a cosmological constant found in A. A. Starobinsky, JETP Lett. 37, 55 (1983). For the $R + R^2$ model, this was proved in A. A. Starobinsky and H.-J. Schmidt, Class. Quant. 4, 695 (1987).

Generic late-time asymptote of classical solutions of GR with a cosmological constant Λ both without and with hydrodynamic matter (also called a Fefferman-Graham expansion):

$$ds^2 = dt^2 - \gamma_{ik} dx^i dx^k$$

$$\gamma_{ik} = e^{2H_0 t} a_{ik} + b_{ik} + e^{-H_0 t} c_{ik} + \dots$$

where $H_0^2 = \Lambda/3$ and the matrices a_{ik}, b_{ik}, c_{ik} are functions of spatial coordinates. a_{ik} contains two independent physical functions (after 3 spatial rotations and 1 shift in time + spatial dilatation) and can be made unimodular, in particular. b_{ik} is unambiguously defined through the 3-D Ricci tensor constructed from a_{ik}. c_{ik} contains a number of arbitrary physical functions (two - in the vacuum case, or with radiation).

Generic initial conditions near a curvature singularity in these models: anisotropic and inhomogeneous (though quasi-homogeneous locally).

1. Modified gravity models (the $R+R^2$ and Higgs ones). Structure of the singularity at $t \to 0$:

$$ds^2 = dt^2 - \sum_{i=1}^{3} |t|^{2p_i}\, a_l^{(i)} a_m^{(i)}\, dx^l dx^m,\ 0 < s < 3/2,\ u = s(2-s)$$

where $s = \sum_i p_i,\ \ u = \sum_i p_i^2$ and $a_l^{(i)},\ \ p_i$ are functions of $\mathbf{r}$. Here $R \propto |t|^{1-s} \to \infty$ (for $1 < s < 3/2$, otherwise it approaches a constant) and $R^2 \ll R_{\alpha\beta}R^{\alpha\beta}$. No infinite number of BKL oscillations.

2. GR model with a very flat potential.

A similar behaviour but with $s = 1,\ \ u < 1$ and with negligible potential.

In both cases, spatial gradients may become important for some period before the beginning of inflation.

Conclusions

- There exists a class of inflationary models having $n_s - 1 = \frac{2}{N}$ and $r \sim \frac{10}{N^2}$ which is most favoured by the Planck and other recent observational data.
- This class includes the one-parametric pioneer $R + R^2$ and Higgs inflationary models in modified (scalar-tensor) gravity, and more general two-parametric models including a GR model with a very flat inflaton potential.
- Inflation is generic in this models.
- Non-Gaussianity of primordial perturbations is small, as in all one-field slow-roll inflationary models.
- The most critical observational test for these models is small, but not too small value of r. The preferred value in the most elegant models with one free parameter is $r = \frac{12}{N^2} \approx 4 \times 10^{-3}$. Well possible to measure in future, in particular, the PRISM project plans to reach $r \sim 3 \times 10^{-4}$ at 3σ.